Propagation of ultra-high energy cosmic rays in magnetic fields

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***Abstract*— In this work, the effects of magnetic fields on the propagation of ultra-high energy cosmic rays were investigated. A cosmic ray propagator was used to simulate a dataset of events, which was analyzed in search of a better understanding of how magnetic fields affect the trajectory and the arrival directions of these particles on Earth's atmosphere. An intricate relation was found between the energy and compostion of the particles and the effects of magnetic fields on their propagation. In order to validate the methodology used, the results were compared to experimental studies of deflections caused by magnetic fields. Similar results were found experimentally and by simulation, which suggests that the model used is an accurate description of the phenomenom.**

I. Introduction

Cosmic rays are the most energetic particles known to science. Due to their extreme energies, this phenomenon played a lead role in the discovery of a myriad of particles, such as positrons [1], muons [2], pions [3], and others. Despite their importance to the development of particle physics and having been discovered over a century ago, there are still a few questions yet to be answered about these remarkably energetic particles, especially concerning their propagation and acceleration methods at the highest energies. The challenge of discovering sources of ultra-high energy cosmic rays relies on two major issues. The first one is the lack of events at the highest energies. As the energy increases, the flux decreases drastically, therefore, making the detection of ultra-high cosmic rays a challenging task. However, the Pierre Auger Observatory has been providing an unprecedented amount of data about the flux at the highest energies, making possible the study of the ultra-high energy component of the cosmic ray spectrum. The second issue is the charge of these particles. Once cosmic rays are charged particles, their paths between their sources and Earth's atmosphere are distorted by galactic and extragalactic magnetic fields, shuffling their arrival directions and, consequently, hindering the identification of their origins. For this reason, the study of the effects of magnetic fields on the propagation and, in particular, on the arrival direction of these particles on Earth's atmosphere is of great relevance in order to further investigate which astronomical objects are accelerating these particles to such energies. The objective of this work is to use the software ExtraGalactic Cosmic Ray Propagator (EGCRProp) [4] to investigate the effects of magnetic fields on cosmic rays propagation at the highest energies, focusing on their arrival direction to an observer on Earth, and compare our findings to results from the Pierre Auger Observatory [5].

II. Methodology

A. Modeling of the magnetic fields

The ExtraGalactic Cosmic-Ray Propagator was used to simulate the propagation of ultra-high energy cosmic rays through magnetic fields. The EGCRProp is an object-oriented C++ program based on the Monte Carlo method that models the propagation of cosmic rays through the extragalactic medium. The code is composed of two parts: a magnetic field tensor generator and a propagator. The program provides not only the position of the particle at each step of the simulation but also dynamical parameters, such as the angular deflection and total distance traveled by the particle.

The extragalactic magnetic fields are modeled as spherical cells with coherent fields. The field's orientation of each cell is random, however, their intensity is constant at $B = 1.0$ nG. From the center of the sphere to a radius of 0.95 Mpc the field is completely coherent, and outside of this region, the field's direction transitions smoothly between cells. The direction of the zenithal or azimuthal angle α of the field is given by the Eq.1.

$$\alpha = \left(\frac{\alpha_1}{d_1} + \frac{\alpha_2}{d_2}\right)\left(\frac{1}{d_1} + \frac{1}{d_2}\right)^{-1}, \tag{1}$$

being α_1 and α_2 the zenithal or azimuthal angles of the two closest cells and d_1 and d_2 the distances to the two closest cells.

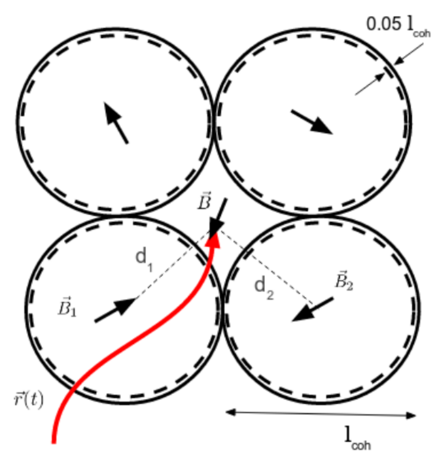

Fig. 1: Schematic diagram of the magnetic field cells and the trajectory of a particle [4].

To find the path followed by the particle through the magnetic fields and its dynamic parameters, the equation of variation of linear momentum of a relativistic particle traveling through a magnetic field is used, shown in Eq.2.

$$\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}), \quad (2)$$

being $\vec{p}$ the particle's relativistic linear momentum, q its eletric charge, $\vec{E}$ the electric field and $\vec{B}$ the magnetic field. In the simulation, electric fields are not considered, therefore the variation of the particle's momentum is only caused by its interaction with magnetic fields.

To obtain the trajectory, the particle's equation of motion is numerically integrated:

$$\vec{v}_f = \vec{v}_0 + \int_0^{\delta t} (\vec{v} \times \vec{\omega}_B) dt, \quad (3)$$

being $\vec{\omega}_B = q\vec{B}/\gamma m$ the cyclotron frequency of the particle and the step chosen for the time intervals must be small ($\delta t/T \lesssim 10^{-3}$) compared to the particle's revolution period $T = 2\pi/\omega_B$.

The options available for the stop condition of the simulation are choosing a maximum distance traveled by the particle, taking into account its tortuous path, or setting a maximum distance that the particle can reach from its starting point, therefore, determining the radius of a spherical shell centered on the source, so that when the particle hits any point of the shell, the simulation is terminated.

B. Simulating the dataset

In order to study the effects of magnetic fields on the propagation of cosmic rays, firstly, we need to generate a dataset of events with mixed composition and different energies. The options available using the EGCProp for the primary cosmic ray are H, He, O, Si and Fe nuclei, and its energy can range from 10^{17} eV to 10^{20} eV. The values of mass and charge of each atomic nuclei used are shown in Table I. For each composition, 1000 events were simulated for each energy starting from the lowest energy possible and multiplying the previous energy by $\sqrt{10}$ until it reaches the upper threshold. In the simulation, the distance to the source is used as a stop condition. To this dataset, the simulation is shut down when the particle reaches a 5 Mpc distance from the source. This value was chosen because it is similar to the distance of the nearest cosmic rays sources candidates to Earth [6].

	H	He	O	Si	Fe
Mass (A)	1	4	16	28	56
Charge (e)	+1	+2	+8	+14	+26

TABLE I: Mass and charge of each atomic nuclei used for the simulations.

C. Analyzing the dataset

1) Path deviation: The first step to analyzing the dataset simulated was to calculate the path deviation suffered by each particle on its trajectory. The path deviation was defined by the equation:

$$\delta = \frac{R - R_0}{R_0}, \quad (4)$$

being R the total distance traveled by the particle, considering the sinuosity of its trajectory, and R_0 the final distance from the source reached by the particle, defined beforehand as 5 Mpc. This is a measure of the tortuosity of the path followed by the particle that, in the absence of magnetic fields, would have traveled in a straight line. In this manner, if the cosmic ray was unaffected by galactic fields, the total distance traveled by the particle would be equal to its final distance from the source, hence, the path deviation would be null.

Using the trajectory provided by the simulator, the path deviation was calculated to each event. The mean and its standard error were calculated to each set of events and a plot of the path deviation as a function of energy was made to each composition, shown in Fig. 2.

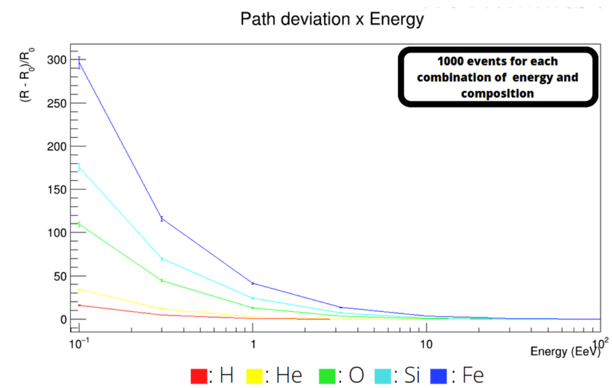

Fig. 2: Path deviation as a function of energy for each composition.

Analyzing the results, it's noticeable the relation between path deviation and energy. The greater the energy of the primary, the lesser is the effect of magnetic fields on its propagation. In other words, as the energy of the particle increases, the more similar to a straight line its trajectory gets. It is also noteworthy the relation between composition and path deviation. Ligther compositions, such as H or He nuclei, tend to be less affected by magnetic fields than heavier composition, such as Si or Fe nuclei. This relation can be explained by the connection between the mass of an atomic nuclei and its charge. The heavier the nuclei, the greater its charge and, therefore, the greater the effect of magnetic fields on its propagation throughout the universe.

2) Angular deviation from the source's perspective: The angular deviation from the perspective of the source caused by magnetic fields on the propagation of cosmic rays was also analyzed. The angular deviation was defined as the angle between the position vector of the particle at the end of the simulation described by a reference frame set on the source and its position vector if it had followed a straight path. The angle deviation was calculated for each event and a graph of the angle deviation as a function of energy was plotted, as shown in Fig. 3.

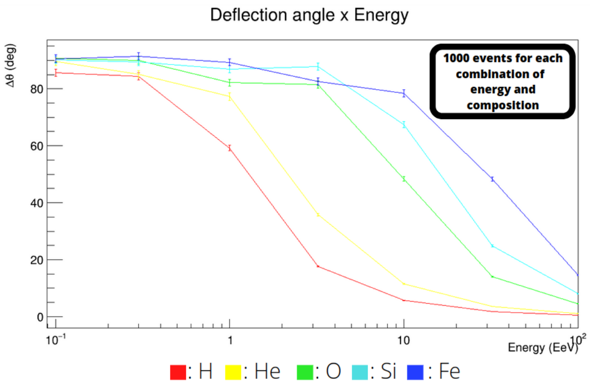

Fig. 3: Angle deviation from the source's perspective as a function of energy.

Similarly to the path deviation analysis, at lower energies and heavier compositions, magnetic fields affect more drastically the propagation of these particles.

3) Angle deviation on the arrival directions: Despite the analysis made until now being important to identify the relation between the effects of magnetic fields on ultra-high energy cosmic rays propagation and energy and composition, the quantities measured in the simulation cannot be quantified empirically. Observatories on Earth cannot determine the total distance traveled by a particle from its source until detection, or calculate the angle deviation from the perspective of the source. What is measurable is the arrival directions of these particles on Earth's atmosphere. Therefore, it is useful to describe the deviation caused by magnetic fields on the arrival directions of the cosmic rays.

To do so, the reference frame, initially centered on the source, must be shifted to the final position of the particle, where it would have been detected. However, using EGCRProp, it is only possible to set the position of the source, while the final position of the particle, the position of the "observer", is determined by the cosmic ray's chaotic path through the magnetic fields randomly generated. This means that the observer's reference frame is different for each event. In order to make all these reference frames equivalent in the perspective of arrival directions, it is convenient to rotate each reference frame, pointing its zenith to the source, which is fixed, as shown in Fig. 4. By doing so, for every single observer, the source will be centered right in the middle of the celestial map, therefore, making them equivalent in terms of angular deviation from the source.

The process of the reference frame shift is described in Fig. 5. First, the reference frame is translated to the final position of the particle. The translation of the reference frame can be mathematically described as:

$$\begin{cases} x' = x - x_f, \\ y' = y - y_f, \\ z' = z - z_f, \end{cases} \tag{5}$$

being (x', y', z') the coordinates of the new reference frame,

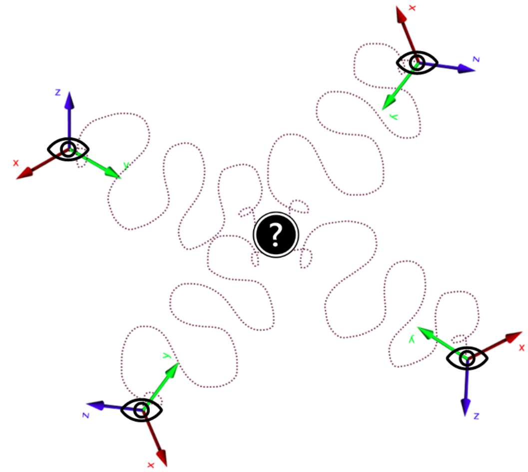

Fig. 4: Illustration of the different reference frames of observers reached by a propagated particle.

(x, y, z) the coordinates of the initial reference frame and (x_f, y_f, z_f) the coordinates of the final position of the particle at the end of the simulation described in terms of the initial reference frame.

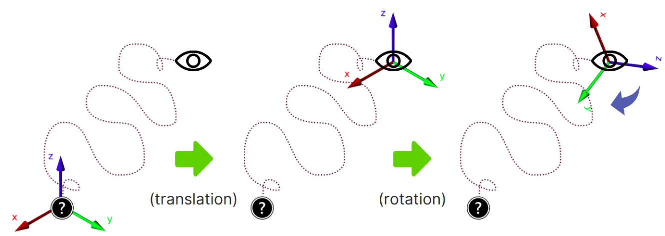

Fig. 5: Process of reference frame shift in order to describe the arrival direction in terms of the observer's perspective.

Then, this reference frame is rotated, pointing its zenith to the source. The rotation of the reference frame can be mathematically described as:

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix},$$

$$\begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} \cos\gamma & \cos\beta\sin\gamma & \sin\beta\sin\gamma \\ -\sin\gamma & \cos\beta\cos\gamma & \sin\beta\cos\gamma \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix},$$

$$\begin{cases} x'' = x'\cos\gamma + y'\cos\beta\sin\gamma + z'\sin\beta\sin\gamma, \\ y'' = -x'\sin\gamma + y'\cos\beta\cos\gamma + z'\sin\beta\cos\gamma, \\ z'' = -y'\sin\beta + z'\cos\beta, \end{cases} \tag{6}$$

being (x'', y'', z'') the coordinates of the rotated reference frame and β and γ the angles rotated around the x and z axis, respectively.

To find the values of β and γ, we can use the fact that the zenith of the rotated reference frame is pointed toward the source. Therefore, the new coordinates of the source should be expressed only in terms of the coordinate y''. Hence, the coordinates x and z in the new coordinate system should be:

$$
\begin{cases} x'' = 0, \\ z'' = 0. \end{cases} \tag{7}
$$

Once the initial reference frame is centered on the source, hence:

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{Eq.5} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -x_f \\ -y_f \\ -z_f \end{bmatrix}
$$

$$
\xrightarrow{Eq.6} \begin{bmatrix} x'' \\ y'' \\ z'' \end{bmatrix} = \begin{bmatrix} -x_f \cos\gamma + y_f \sin\gamma \\ -\cos\beta(x_f \sin\gamma + y_f \cos\gamma) + z_f \sin\beta \\ -\sin\beta(x_f \sin\gamma + y_f \cos\gamma) - z_f \cos\beta \end{bmatrix}
$$

$$
\xrightarrow{Eq.7} \begin{cases} 0 = -x_f \cos\gamma + y_f \sin\gamma, \\ 0 = -\sin\beta(x_f \sin\gamma + y_f \cos\gamma) - z_f \cos\beta, \end{cases}
$$

$$
\rightarrow \begin{cases} \gamma = \arctan\left(\frac{x_f}{y_f}\right), \\ \beta = \arctan\left(\frac{z_f}{-x_f \sin\gamma - y_f cos\gamma}\right), \end{cases} \tag{8}
$$

being these the angles necessary to point the zenith of the reference frame to the particle's source.

Using Eqs. 6 and 8, the particle's trajectory can be described in terms of the new frame of reference of the observer.

To evaluate the angular deviation caused by magnetic fields, the arrival direction of the particles must be compared to the position of the source on the sky. The definition of arrival direction chosen was the direction of the particle's position relative to the observer one iteration before the end of the simulation. Describing these directions in the new coordinates, we can plot a sky map of the arrival directions, in which any variation from the zenith is the angular deviation caused by magnetic fields on the particle's propagation. However, in order to plot the sky map, we must, first, transform the Cartesian coordinates of our new reference frame into spherical coordinates:

$$
\begin{cases} r = \sqrt{x''^2 + y''^2 + z''^2}, \\ \theta = \arccos\left(\frac{z''}{r}\right), \\ \varphi = \arctan\left(\frac{x''}{y''}\right). \end{cases} \tag{9}
$$

And then into celestial coordinates:

$$
\begin{cases} \delta = 90° - \theta, \\ \alpha = \varphi. \end{cases} \tag{10}
$$

Lastly, we can plot the arrival directions in celestial coordinates, as shown in Fig. 6.

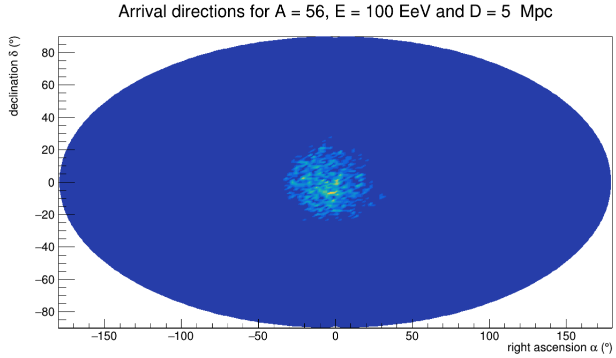

Fig. 6: Celestial map of arrival directions of 1000 Fe nuclei with energy of 100 EeV arriving from a source at 5 Mpc away.

The events' arrival directions are distributed around the center of the map. If the particles had traveled in a straight line, in the absence of magnetic fields, all the events should lie at the zenith. The deviations seen are caused by the effects of magnetic fields on cosmic rays propagation and, therefore, can be used to infer the expected deflection caused by magnetic fields on the arrival directions of these particles.

To quantify the angular deviation seen by the observer, it can be defined as:

$$
\Delta\theta_{obs} = \sqrt{(\alpha - \alpha_s)^2 + (\delta - \delta_s)^2}, \tag{11}
$$

being (α, δ) the arrival direction of the particle and (α_s, δ_s) the direction of the source in the sky. Since the source in the direction of the zenith ($\alpha_s = 0$ and $\delta_s = 0$), we get:

$$
\Delta\theta_{obs} = \sqrt{\alpha^2 + \delta^2}, \tag{12}
$$

The angular deviation measured by the observer was calculated for each event and the mean and its standard deviation were calculated for each set of events (1000 particles for each combination of energy and composition). The results are shown in Fig. 7.

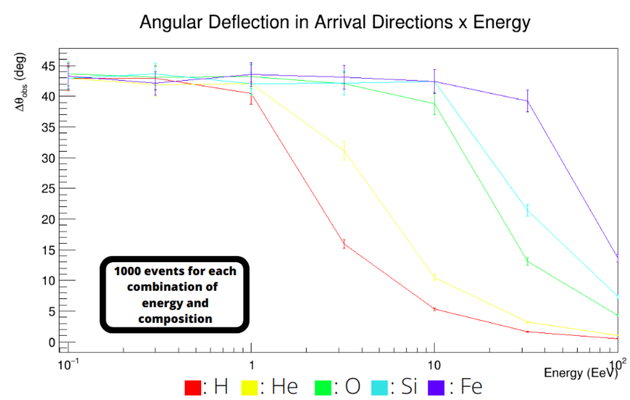

Fig. 7: Angular deflection measured by an observer 5 Mpc away from a source as a function of the particle's energy for different compostions.

As seen before, the effects of magnetic fields on the propagation of cosmic rays drop drastically as energy increases, as well as for lighter compositions. It is also important to point out that for extremely energetic particles, especially for low-charged compositions, the effects of magnetic fields are small enough that is possible to extract information out of the arrival directions of these particles about the direction of their sources.

One possible issue that might arise from this method is related to using the distance to the source as a breakpoint to the propagation of the particle. Since when the particle reaches a certain distance previously determined, the simulation stops and the event is ”detected” by the observer, the particle can’t have come from the opposite direction of the source, because it would have been at a greater distance from the source than the distance determined as the breakpoint. Therefore, the use of this method limits the angular deviation to only one-half of the entire celestial sphere. However, as shown in Figs. 2 and 3, particles at the highest energies travel practically in a straight line, and the angular deflection caused by magnetic fields is much lower than the artificial limit generated by our method, therefore, making it useful to analyze the effects of magnetic fields on the propagation of ultra-high energy cosmic rays.

4) Comparison to experimental results from the Pierre Auger Observatory: To validate the method used to study the effects of magnetic fields on ultra-high energy cosmic rays, it is interesting to compare the results from simulation to experimental studies.

In a study carried out by the Pierre Auger Collaboration, the correlation between the flux of ultra-high energy protons and nearby active galactic nuclei (AGNs), which are a candidate for possible sources of cosmic rays at the highest energies, was investigated. Using the first data set taken by the Pierre Auger Observatory, the parameters analyzed were the maximal AGN distance, the minimal particle’s energy, and the maximal angular deviation. By setting these parameters to 75 Mpc, 57 EeV, and 3.1°, respectively, it was found that the probability of the correlation resulting from an isotropic distribution of arrival directions was rejected with 99% of confidence level [7].

To assess the validity of the results of this current work, the parameters used by the Pierre Auger Collaboration can be inserted into the model used and compare the angular deviation predicted by the simulation to the maximal correlation angular deviation found empirically using the Pierre Auger Observatory’s data. To accomplish that, first, we must distribute the sources according to the maximal distance of 75 Mpc. The distance of the source is equivalent in our model to the distance traveled by the particle, which is the conditional breakpoint of the simulation. Therefore, to distribute the sources within 75 Mpc in our model, we generate a random number between 0 and 75 Mpc using a uniform distribution to use it as the stop condition of the simulation. The composition of particles emitted by the source is H nuclei and their energy range from 57 EeV up to 100 EeV. However, the probability of an event with energy E must follow the power law that describes the energy spectrum of cosmic rays, shown in Fig. 8.

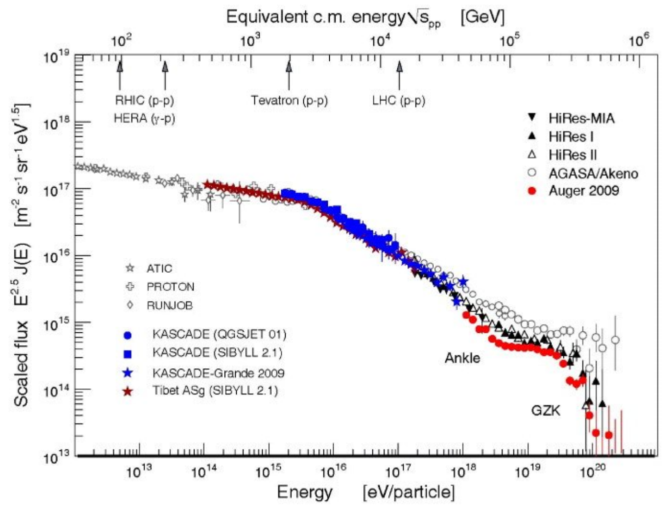

Fig. 8: Cosmic ray energy spectrum measured by different experiments [8].

The energy spectrum power law is described by the equation:

$$\frac{dN}{dE} \propto E^{-\alpha}, \tag{13}$$

being N the flux of events, E the energy, and α the spectral index, whose value changes along different regions of the spectrum.

In the region of the spectrum above $\sim 5 \times 10^{19}$ eV, known as the GZK cutoff, there is a drastic suppression of the flux due to the interaction of cosmic rays with photons from the cosmic background radiation. The spectral index of this region is measured to be 5.1 ± 0.7 and, since the particles analyzed have energies above 57 EeV, this will be the spectral index used for the probability function of energies. A histogram of the distribution of energies generated is shown in Fig. 10.

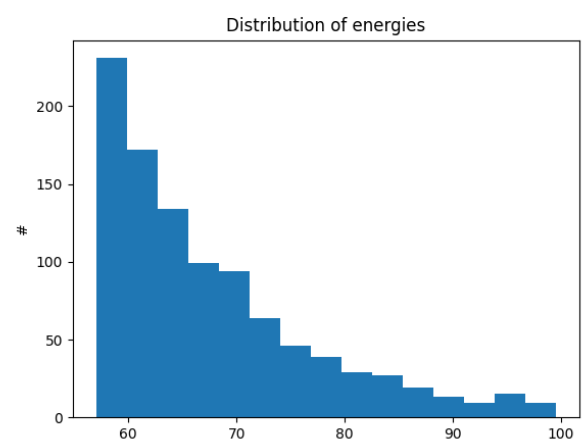

Fig. 9: Histogram of the distribution of 1000 energies randomly generated following a power law with spectral index $\alpha = 5.1$.

Using the conditions previously described, 1000 events were simulated. Their arrival directions are shown in Fig.

10 and the angular deviation measured by the observer was calculated for each one of them. The mean and its standard deviation were calculated to be $3.1° \pm 0.3°$, which agrees with the findings from the Pierre Auger Observatory.

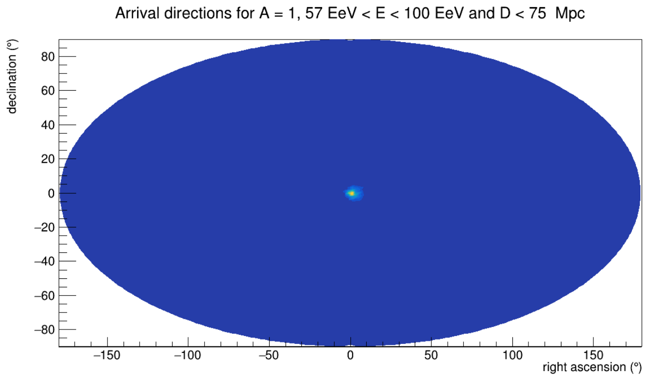

Fig. 10: Celestial map of arrival directions of 1000 H nuclei with energies ranging from 57 EeV to 100 EeV arriving from sources within 75 Mpc.

This result suggests that the modeling made for magnetic fields and cosmic rays propagation is an adequate description of the situation and can be used to further investigate the effects of magnetic fields on the propagation of these particles.

III. Conclusions

This work addresses the effects of magnetic fields on cosmic ray propagation at the highest energies. The EGCProp simulator was used to create a dataset of trajectories of particles through extragalactic magnetic fields for different combinations of energy and composition.

The trajectories of the particles were analyzed in search of connections between the effects of magnetic fields and their energy and composition. It was found, as expected, that the higher the energy of the particle, the lesser the deflection caused by magnetic fields on their trajectory. In this way, at the highest energies, cosmic rays travel practically in a straight line between their source and detection. Therefore, for ultra-high energy cosmic rays, it is possible to extract information about the direction of their sources from the flux measured on Earth. A similar relation was observed for composition. The effects of magnetic fields were greater on heavier compositions than on lighter ones. This can be explained by the relation between the mass and charge of atomic nuclei. Since heavier compositions have also a greater charge, magnetic fields have stronger effects on their trajectory.

The angular deviation in the arrival directions of particles relative to the direction of their sources was also analyzed. To do so, the reference frame centered on the source in which the trajectory was being described had to be shifted to describe it from the observer's reference frame. The angular deviations on arrival directions were calculated for the dataset and similar results were observed for the relation between angular deviations on arrival directions and energy and composition.

In order to validate the methodology used to analyze the effects of magnetic fields on arrival directions, the results were compared to findings made by the Pierre Auger Observatory on the angular deviation on the arrival direction from the position of possible sources on the sky. While the experimental study made by the Pierre Auger Observatory found a $3.1°$ angular deviation caused by magnetic fields, the methodology described in this work arrived at a similar result of $3.1° \pm 0.3°$, therefore, agreeing within the margin of error, which suggests that the model used describes satisfactorily the situation and is a viable tool to further investigate the effects of magnetic fields on ultra-high energy cosmic rays.

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IV. Acknowledgements

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