Black Hole Thermodynamics

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The Theory of Thermodynamics, which was developed in the 19th century, before the advent of General Relativity and Quantum Mechanics, plays a central role in the study of the properties of ordinary macroscopic matter. Black Holes, originally study objects of General Relativity, surprisingly can behave as thermodynamic entities that have temperature and entropy. The geometric formulation of General Relativity is discussed, along with the comparison between the Mechanical Laws of Black Holes and the Laws of Thermodynamics. Lastly, are mentioned problems that arise with the classical approach and how considering quantum effects is important to achieve a better description of black hole states.

I. INTRODUCTION

General Relativity (GR) is one of the most successful theories in describing the physical world. It describes gravity as a consequence of the spacetime geometry. The fact that gravity is attractive explains why stars and galaxies were formed and became stable, as their equilibrium is maintained as a result of the balance between gravity and thermal pressure or rotation and internal motions. In a situation where the balance can no longer be sustained, the object begins to shrink. It reaches a critical size such that the gravitational collapse is inevitable. The "gravitational field" becomes so strong that not even light can escape past a region called event horizon. This extreme object is called a black hole, and GR predicts that it has a singularity lying inside. The Cosmic Censorship Conjecture ensures that singularities must be trapped inside black holes, and so they are hidden from external observers, in a way that the breakdown of predictability would not affect the external spacetime [1].

Furthermore, it has been shown that a collapsing body loses information, turning into a stationary black hole that can be described by only a few parameters. One can derive expressions for a stationary axisymmetric black hole, and two quantities that appear in the expressions, the surface gravity κ and the area of the event horizon A, can be related to the thermodynamic quantities temperature T and entropy S, respectively [2]. The analogies suggest a correspondence between the Laws of Black Hole Mechanics and the Laws of Thermodynamics, giving rise to Black Hole Thermodynamics. In this work are used units such that $c = \hbar = 1$.

II. PHYSICS IN CURVED SPACETIME

Spacetime is a pair $(M, g_{\mu\nu})$ where M is a 4-dimensional manifold and $g_{\mu\nu}$ is a Lorentzian metric. The Laws of Physics can be generalized to curved spacetime via the minimal-coupling principle, taking valid laws in inertial coordinates in flat spacetime and writing them in a tensorial form [3]. With that purpose, the motion of a freely-falling particle along a parameterized path $x^\mu(\lambda)$ is given by:

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0, \tag{1}$$

where $\Gamma^\mu_{\rho\sigma}$ is the metric connection and λ is an affine parameter. In GR an unaccelerated test particle will follow a timelike geodesic, the path that maximizes proper time and obeys (1). It is necessary to describe the energy and momentum of a system of particles. This is done by means of the energy-momentum tensor $T^{\mu\nu}$. The local conservation law of energy-momentum turns out to be in curved spacetime:

$$\nabla_\mu T^{\mu\nu} = 0, \tag{2}$$

where ∇_μ is the covariant derivative. An observer with four-velocity v^μ will measure an energy density $T_{\mu\nu}v^\mu v^\nu$, and the Weak Energy Condition states that it must be non-negative [4].

Einstein field equation (3) describes how matter is affected by the curvature of spacetime, which manifests itself as gravity, and how the curvature is affected by the presence of energy and momentum.

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right) \tag{3}$$

$R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ the metric of the spacetime and G Newton's constant of gravitation.

Einstein's equation is a set of six independent second-order and non-linear partial differential equations for the metric $g_{\mu\nu}$. Although they are usually very difficult to solve, there are a few exact solutions in special cases where the spacetime possesses certain symmetries. Of fundamental interest is the case in which $T_{\mu\nu} = 0$, leading us to the vacuum Einstein's equation:

$$R_{\mu\nu} = 0. \tag{4}$$

Within the solutions of such equations are the rotating black holes.

III. CONFORMAL DIAGRAMS AND CAUSALITY

The classical and hyperbolic nature of GR and Einstein's equation guarantees that, given the initial conditions of a system, one could predict its future state. Not only in special relativity one must move forward in time, but also the speed of light puts a limit on how fast something can move. As a result, any signal must stay inside the light cone of each event.

Given a spacetime with a specific metric $g_{\mu\nu}$, it is reasonable to portrait its causal structure (defined by the light cones) as a whole. A conformal diagram is a diagram of a spacetime with the metric in a clever coordinate system. One works with a timelike coordinate and a spacelike one, and the radial light cones are straight lines just like in Minkowski spacetime [3]. Futhermore, one brings infinity to some finite value of coordinates, in order to capture global properties and then the causal structure.

Null curves remain invariant under conformal transformations, i.e., under local change of scale. One can perform this by multiplying the original metric by some nonzero function dependent of spacetime $g_{\mu\nu} \to \omega^2(x)\; g_{\mu\nu}$. There is in flat spacetime a coordinate transformation that permits us to see the whole spacetime at once, and leads us to a metric that is conformally related to the Minkowski one. Such coordinates, T and R, have ranges:

$$0 \leq R < \pi, \quad |T| + R < \pi, \tag{5}$$

and they are related to the Minkowski metric

$$ds^2 = -dt^2 + dr^2 + r^2\; d\Omega^2 \tag{6}$$

via

$$ds^2 = \omega^{-2}(T,R)\left(-dT^2 + dR^2 + \sin^2 R\; d\Omega^2\right), \tag{7}$$

where $\omega(T,R) = \cos T + \cos R$. The result is the conformal diagram in figure 1.

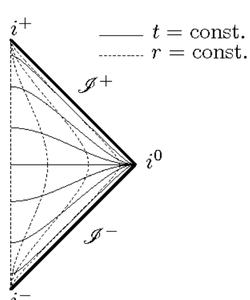

Figure 1: Conformal diagram for flat spacetime [5]

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The format of the diagram allows us to subdivide the conformal infinity into the following regions:

$$\begin{aligned} i^+ &= \text{future timelike infinity} \\ i^0 &= \text{spatial infinity} \\ i^- &= \text{past timelike infinity} \\ \mathcal{I}^+ &= \text{future null infinity} \\ \mathcal{I}^- &= \text{past null infinity} \end{aligned}$$

Asymptotically flat spacetimes share the structure of $\mathcal{I}^+$, i^0, and $\mathcal{I}^-$ with Minkowski spacetime [3].

IV. SCHWARZSCHILD BLACK HOLES

Given Einstein's equation, a next step is to analyze the case of a spherically symmetric "gravitational field", which is, with good approximation, the type generated by planets and stars, for example. The spherically symmetric vacuum solutions are automatically static by the Birkhoff's theorem in GR; the solution is unique and given by the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{8}$$

where M is interpreted as the mass of the object that generates the "gravitational field" [3].

The curvature of spacetime can manifest itself in the twisting of light cones. One can find the radial null curves by setting the angular coordinates constant and $ds^2 = 0$. Doing this in (8), one finds:

$$\frac{dt}{dr} = \pm\left(1 - \frac{2GM}{r}\right)^{-1}. \tag{9}$$

Two things must be highlighted. First, as $r \to \infty$, $dt/dr \to 1$, which tells us the metric (8) is asymptotically flat. Second, as $r \to 2GM$, $dt/dr \to \pm\infty$. This shows an apparent incapacity to reach the Schwarzschild radius $r_S = 2GM$. An inertial observer at infinity would see something near r_S moving more and more slowly, with time, toward r_S, with increasing redshift. This is an artifact of coordinate system choice; an object near r_S can, in fact, cross r_S in a finite amount of proper time.

To see this, the tortoise coordinate is introduced:

$$r^* = r + 2GM\; ln\left(\frac{r}{2GM} - 1\right), \tag{10}$$

as well as coordinates naturally adapted to the null geodesics:

$$v = t + r^* \qquad u = t - r^*, \tag{11}$$

so that $v = const$ describes ingoing null geodesics, and $u = const$ outgoing ones. By choosing to use the original coordinate r and the new coordinate v, one finds the metric in the ingoing Eddington-Finkelstein coordinates:

$$ds^2 = -\left(1 - \frac{2GM}{r}\right)dv^2 + (dv\; dr + dr\; dv) + r^2 d\Omega^2. \tag{12}$$

The light cones are well-behaved in r_S and this hypersurface is at a finite coordinate value. Furthermore, it acts as a no-return point, so a particle that crosses the Schwarzschild radius can never come back. The hypersurface $r = r_S$ is an event horizon. An observer outside r_S cannot receive any type of information from the inside of r_S.

The Schwarzschild solution must be extended to points $r < r_S$, since real particles can cross the event horizon. For that matter are defined the following coordinates:

$$\begin{cases} T = \frac{1}{2}(v' + u') \\ R = \frac{1}{2}(v' - u') \end{cases}, \tag{13}$$

with range

$$-\infty \leq R \leq \infty, \quad T^2 < R^2 + 1, \tag{14}$$

and where $v' = e^{v/4GM}$ and $u' = -e^{-u/4GM}$. Therefore, the metric becomes:

$$ds^2 = \frac{32G^3M^3}{r} e^{-r/2GM}(-dT^2 + dR^2) + r^2 d\Omega^2, \tag{15}$$

with r defined implicitly by T and R.

The coordinates (T, R, θ, ϕ) are the Kruskal coordinates. The Kruskal solution (15) is the analytic extension of the Schwarzschild solution (8), even though it is not complete since it has an intrinsic singularity at $r = 0$ [6]. The global structure of the spacetime is represented in the conformal diagram of figure 2.

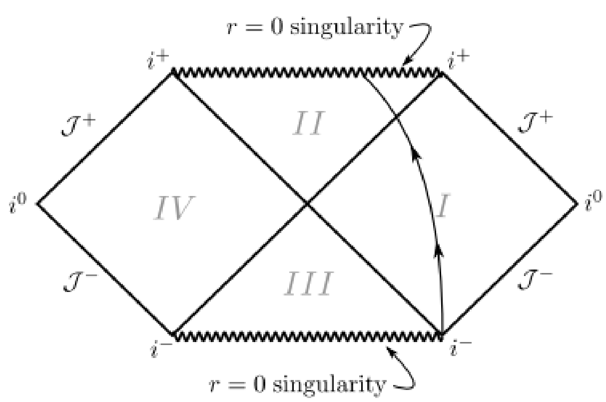

Figure 2: Conformal diagram for Schwarzschild spacetime [7]

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The maximally extended Schwarzschild solution not only describes a black hole, but also a white hole and another asymptotically flat region, connected to our spacetime by a wormhole [3]. It is a very idealized situation, and the presence of matter anywhere in this spacetime could drastically change its properties.

V. MORE GENERAL BLACK HOLES

One looks for spherically symmetric vacuum solutions for Einstein's equation not only for mathematical reasons, but also because there are in nature massive objects with such a symmetry. Planets, for example, have an irregular surface and so one has to take into account other parameters in order to have a better description of the exact field. It can be done by writing the metric in multipole terms, leading to a very difficult calculation. For black holes the story is different. There are established theorems [8] that state that the external "gravitational field" of a black hole reduces to that of a Kerr-Newman black hole, described by only three parameters: mass, charge, and angular momentum [9–11]. This is known as the no hair theorem. In such a case one has stationary asymptotically flat solutions, as one is describing the final stage of a gravitational collapse and the gravitational influence must be negligible infinitely far away from the black hole. So, a conformal diagram of this type of black hole will share $\mathcal{I}^+$, i^0, and $\mathcal{I}^-$ with flat spacetime, just like in figure 3.

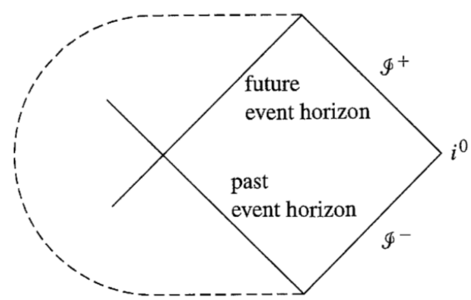

Figure 3: A generic conformal diagram for an asymptotically flat spacetime with an event horizon [3]

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The Cosmic Censor Conjecture states that a complete gravitational collapse of a body never results in a naked singularity, but rather in a singularity "protected" by an event horizon, i.e., a black hole [8]. Assuming the Weak energy condition and the Cosmic Censor Conjecture, it has been found [2] that in an asymptotically flat spacetime the area of a future event horizon is nondecreasing. This is Hawking's area theorem.

An event horizon is the boundary between events that can influence $\mathcal{I}^+$ from those that cannot. $J^-(S)$ is the set of points in the spacetime that can be reached from S through past directed causal paths. One can define an event horizon as the boundary of $J^-(\mathcal{I}^+)$[3]. The event horizon is a null hypersurface, so it has a null normal vector that is also tangent to it. One can enquire how, given a spacetime metric, find if there is any event horizon. It is fairly simple to find a candidate for a horizon if the coordinates were chosen wisely, in a way that the type of hypersurfaces changes from timelike to null for some fixed $r = r_H$. In such a case the norm $g^{rr} = g^{\mu\nu}(\partial_\mu r)(\partial_\nu r)$

becomes zero, where $\partial_\mu r$ is a dual-vector that is normal to the hypersurfaces. Of course, in Schwarzschild $g^{rr} = 1 - 2GM/r$, and so $r_H = 2GM$, as we already know. Charged and rotating black holes are described by solutions that are more commonly presented in coordinates adapted to the event horizons.

Right know the main interest is to describe a rotating black hole, with a symmetry around an axis of rotation, and it will be considered stationary solutions (the case that there is a timelike Killing vector). The Kerr metric (16) describes a stationary, axissymetric vacuum solution of Einstein's equation (3).

$$ds^2 = \left\{ -\left(1 - \frac{2GM\,r}{\rho^2}\right) dt^2 - \frac{2GMar\sin^2\theta}{\rho^2}(dt\ d\phi + d\phi\ dt) + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right] d\phi^2 \right\} \quad (16)$$

Where $\Delta(r) = r^2 - 2GMr + a^2$ and $\rho^2(r,\theta) = r^2 + a^2\cos^2\theta$. In this case, the parameters are M (mass of the black hole) and a (angular momentum per unit mass). Replacing $\{2GMr\}$ for $\{2GMr - G(Q^2 + P^2)\}$ in (16) one finds the Kerr-Newman metric, which describes a rotating and charged black hole. Q is the total electric charge, and P the total magnetic one.

Isometries are symmetries of the metric. If the metric is independent of some coordinate σ_*, then the Killing vector $K = \partial_{\sigma_*}$ generates the isometry. In this case,

$$\partial_{\sigma_*} g_{\mu\nu} = 0 \quad \Rightarrow \quad \frac{dp_{\sigma_*}}{d\tau} = 0, \quad (17)$$

where p_μ is the momentum of a particle following a timelike geodesics and τ is the proper time of the particle. So, related to this simmetry, there is a conserved quantity. It is possible to write in terms of components $K^\mu = (\partial_{\sigma_*})^\mu = \delta^\mu_{\sigma_*}$, so $p_{\sigma_*} = K^\nu p_\nu = K_\nu p^\nu$. One can write (17) in a covariant manner as:

$$\nabla_{(\mu}K_{\nu)} = 0 \quad \Rightarrow \quad p^\mu \nabla_\mu (K_\nu p^\nu) = 0. \quad (18)$$

The left hand side equation is Killing's equation, which Killing vector fields satisfy. A stationary solution can have the coordinates adapted in a way that the metric components are time independent, and so the metric has a Killing vector ∂_t, asymptotically timelike. If there is a Killing vector field $\mathcal{X}^\mu$ that is null over a null hypersurface Σ, then Σ is a Killing horizon of $\mathcal{X}^\mu$. If the spacetime is stationary, it has a Killing vector $\mathcal{X}^\mu = K^\mu + \Omega_H R^\mu$ for some constant Ω_H, where $K^\mu = (\partial_t)^\mu$ and $R^\mu = (\partial_\phi)^\mu$. ∂_ϕ is the Killing vector associated with the axial symmetry. In the case of static black holes, the event horizon is a Killing horizon for K^μ. Hawking showed that any stationary black hole must have its event horizon as a Killing horizon for some $\mathcal{X}^\mu$ [3].

The Boyer-Lindquist coordinates (t, r, θ, ϕ) used in (16) are adapted to the horizon, so one can find the event horizon location by calculating $g^{rr} = 0$. It is zero when

$$\Delta(r) = r^2 - 2GMr + a^2 = 0. \quad (19)$$

The situation can be divided into three cases: (1) $GM > a$, (2) $GM = a$ and (3) $GM < a$. The first one has more physical interest, since the second one represents a very unstable situation and the third one leads to a naked singularity. Solving (19) one finds two null hypersurfaces that represent distinct horizons:

$$r_\pm = GM \pm \sqrt{G^2M^2 - a^2}. \quad (20)$$

r_+ represents the outer horizon, an event horizon, and r_- the inner horizon, which is a Cauchy horizon. Even though they are not Killing horizons for $K = \partial_t$, it is useful to calculate the norm:

$$K^\mu K_\mu = -\frac{1}{\rho^2}(\Delta - a^2\sin^2\theta), \quad (21)$$

which is positive at $r = r_+$, meaning that the Killing vector is already spacelike at the outer horizon, except when $\theta = 0, \pi$, where it is null [3]. The set of points where K_μ is null constitutes the stationary limit surface, and the region between it and the outer event horizon is called ergosphere. It is possible to enter and exit the ergosphere, but not to remain stationary inside it. One finds the singularity by setting $\rho = 0$, because in this case the curvature scalar diverges. In this situation, $r = 0$ and $\theta = \pi/2$. As a result of r not being a ordinary radial coordinate, this singularity is a ring in space. Figure 4 shows the structure of a Kerr black hole.

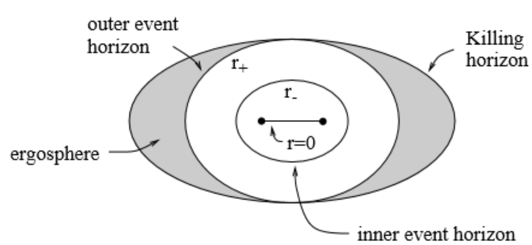

Figure 4: Horizon structure of the Kerr Black Hole [12]

VI. BLACK HOLE THERMODYNAMICS

A triumph of Thermodynamics is to describe the state of macroscopic systems in equilibrium by a few parameters such as volume, temperature and number of particles. As already seen, stationary black holes are described by a few parameters: mass, charge and angular momentum. Additionally, one can make an analogy not only between thermodynamics quantities and the ones that are related to black holes, but also between the Laws of Thermodynamics and the Laws of Black Hole Mechanics.

A. The Penrose Process

One can inquire whether it is possible or not to extract energy from a black hole. Classically, energy cannot be extracted from a nonrotating, neutral (Schwarzschild) black hole [13], since, in this case, the energy consists only of its mass. Given the Kerr metric (16), consider a particle moving along a geodesic with the following 4-momentum, energy, and angular momentum:

$$p^\mu = m\frac{dx^\mu}{d\tau}, \quad E = -K_\mu p^\mu, \quad L = R_\mu p^\mu.$$

The vectors K^μ and p^μ are both timelike at infinity and the energy is necessarily positive. Since K^μ becomes spacelike when one crosses the stationary limit surface, one can wonder if there could exist any particle with $E < 0$ in the ergosphere. Imagine that you are moving outside the ergosphere in a geodesic while holding a heavy rock, and the system (you plus the rock) have 4-momentum $p^{(0)\mu}$. When you reach the ergosphere, you throw the rock into the black hole and, by conservation of momentum,

$$p^{(0)\mu} = p^{(1)\mu} + p^{(2)\mu}, \tag{22}$$

where $p^{(1)\mu}$ and $p^{(2)\mu}$ are your and the rock's 4-momentum, respectively, after the fall. It is possible to arrange the rock's fall in such a way that its energy $E^{(2)} = -K_\mu p^{(2)\mu}$ is negative. Penrose showed [3] that it is also possible to throw the rock into the black hole and then follow a geodesic back outside the ergosphere, so in the end you would necessarily have an energy $E^{(1)} = -K_\mu p^{(1)\mu}$ that is positive. Since the energy conservation must hold

$$E^{(0)} = E^{(1)} + E^{(2)}, \tag{23}$$

after all you come back with more energy than when you entered ($E^{(1)} > E^{(0)}$). Energy does not come from nowhere. In this case it is extracted from the black hole, decreasing its angular momentum as you throw your rock against the rotation flow. This process of energy extraction is called The Penrose Process, and it increases the black hole area as you add mass to it and decrease its angular momentum J. The process has its maximum efficiency when the black hole area remains unaltered [13].

Since the Kerr black hole is stationary, it has a Killing vector field $\mathcal{X}^\mu = K^\mu + \Omega_H R^\mu$ that becomes null at the outer event horizon. Therefore, if a particle with 4-momentum $p^{(2)\mu}$ crosses the event horizon in a future directed path,

$$p^{(2)\mu}\mathcal{X}_\mu < 0 \;\Rightarrow\; L^{(2)} < \frac{E^{(2)}}{\Omega_H}, \tag{24}$$

and, considering the variations of the black hole mass and angular momentum $\delta M = E^{(2)}$, $\delta J = L^{(2)}$,

$$\delta J < \frac{\delta M}{\Omega_H}. \tag{25}$$

B. The Laws of Black Hole Mechanics and of Thermodynamics

There is related to the outer event horizon, a Killing horizon for $\mathcal{X}^\mu$, a function κ named surface gravity, which satisfies:

$$\nabla^\mu(\mathcal{X}^\nu\mathcal{X}_\nu) = -2\kappa\mathcal{X}^\mu. \tag{26}$$

The dominant energy condition incorporates the assumption that massive objects should follow timelike or null world lines [4]. The validity of this condition implies the Zeroth Law of Black Hole Mechanics: the surface gravity κ must be uniform over the event horizon of a stationary black hole [14]. This statement can be associated with the Zeroth Law of Thermodynamics: the temperature T is uniform over a system in thermal equilibrium [1].

To calculate the area of the outer event horizon, one needs to find the induced metric γ_{ij} on it, which is the expression (16) with $r = r_+$, $dt = 0$ and $dr = 0$, so i, j run over $\{\theta, \phi\}$. The result is:

$$\gamma_{ij} = (r_+^2 + a^2\cos^2\theta)\, d\theta^2 + \left[\frac{(r_+^2 + a^2)^2\sin^2\theta}{r_+^2 + a^2\cos^2\theta}\right] d\phi^2. \tag{27}$$

The area is given by the integral:

$$A = \int \sqrt{|\gamma|}\; d\theta\; d\phi = \int (r_+^2 + a^2)\sin\theta\; d\theta\; d\phi, \tag{28}$$

and, then, is

$$A = 4\pi(r_+^2 + a^2). \tag{29}$$

Defining the irreducible mass by

$$M_{irr}^2 = \frac{A}{16\pi G^2}, \tag{30}$$

its differentiation is:

$$\delta M_{irr} = \frac{a}{4GM_{irr}\sqrt{G^2M^2 - a^2}}(\Omega_H^{-1}\delta M - \delta J). \tag{31}$$

Equations (25) and (31) imply $\delta M_{irr} > 0$. So, from the definition (30), it is straightforward that the area of a stationary black hole is nondecreasing with time. This is a special case of Hawking's area theorem, or the Second Law of Black Hole Mechanics [15]. It was deducted for stationary black holes, but it is valid for any type of black hole. Moreover, if the fusion of two distinct black holes happens, then the area of the resulting black hole is greater than the sum of the two areas of the previous ones [1, 15]. Such results lead to a correspondence between the area theorem and the Second Law of Thermodynamics, which states that the entropy of an isolated system can never decrease.

From (30), (31), and considering the surface gravity

$$\kappa = \frac{\sqrt{G^2M^2 - a^2}}{2GM(GM + \sqrt{G^2M^2 - a^2})}, \quad (32)$$

one obtains

$$\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega_H \delta J. \quad (33)$$

The term $\Omega_H \delta J$ in (33) can be considered as the work done on the black hole by throwing matter in it [3]. The expression (33) features a similar structure with the First Law of Thermodynamics:

$$\delta E = T\delta S - P\delta V, \quad (34)$$

so it is called the First Law of Black Hole Mechanics. The following identifications are made:

$$\begin{aligned} E &\leftrightarrow M \\ S &\leftrightarrow A/4G \\ T &\leftrightarrow \kappa/2\pi \end{aligned} \quad (35)$$

In summary, the Laws of Black Hole Mechanics can be related to the ones of Thermodynamics as follows:

Zeroth Law of Thermodynamics:

The temperature T is uniform over a system in thermal equilibrium

Zeroth Law of Black Hole Mechanics:

The surface gravity κ is uniform over the event horizon of a stationary black hole

First Law of Thermodynamics:

$$\delta E = T\delta S - P\delta V$$

First Law of Black Hole Mechanics:

$$\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega_H \delta J$$

Second Law of Thermodynamics:

$$\delta S \geq 0$$

Second Law of Black Hole Mechanics:

$$\delta A \geq 0$$

Classically, the third identification in (35) is inconsistent. One cannot associate a non-zero temperature to a black hole, since nothing can get out of it and, therefore, it does not emit radiation. But, when taking into account Quantum Mechanics, the association becomes valid because of Hawking's radiation, which associates a well defined temperature to the black hole.

Given the expression (32) one can see that it satisfies $\kappa = 0$ for $GM = a$, the unstable "extreme" Kerr black hole. The black hole has a non-zero area, thus there isn't for black holes an analogue of the third law of thermodynamics, considering the formulation which asserts that $S \to 0$ as $T \to 0$ [14].

In order to establish a universal second law, Bekenstein defined the total entropy of the universe as the sum $S_{bh} + S$, where $S_{bh} = A/4G$ is the black hole entropy and S the entropy of everything in the universe that is outside the event horizon. The Generalized Second Law of Thermodynamics (GSL) states that the total entropy never decreases with time:

$$\delta(S + A/4G) \geq 0. \quad (36)$$

One can imagine a box with matter inside that is hanging by a massless rope near a black hole. Because of its content, it carries energy and entropy. The box can be opened at any chosen position, letting the matter fall into the black hole. The more the box approaches the event horizon, the lower will be the energy measured by an observer at infinity, since the value depends on the redshift factor. Opening the box arbitrarily close to the event horizon leads to an imperceptible increase of the black hole area/entropy. In this situation, the universal variation of entropy will be equal to the variation of entropy outside the black hole, which is negative, since the matter previously inside the box gets lost inside the black hole. After the theoretical discovery of Hawking's radiation, it was shown that a static observer measures a radiation around the black hole with a temperature that depends on the distance to the event horizon and that, consequently, creates a pressure gradient. This is responsible for a "buoyancy force" on the box that counterbalances the gravitational force at a point that is not arbitrarily close to the event horizon. It follows that there exists a minimum energy that is dropped into the black hole, and then a minimum increase of the horizon area. Finally, it leads to a positive variation of the total entropy, and hence the GSL is not violated [16].

VII. CONCLUSION

The geometric aspect of General Relativity and its power to describe the nature of spacetime were discussed, as well as the solutions of Einstein's equation that describe black holes, regions in spacetime where gravity is so strong that even light can not get out. Stationary black holes are totally described by mass, total charge, and angular momentum, and energy can be extracted from them by a classical process. Finally, the Laws of Black Hole Mechanics have an analogy with the Laws of Thermodynamics that, classically, has some inconsistencies which are fixed considering Quantum Mechanics. The study of Black Hole Thermodynamics shows an important connection between General Relativity, the Quantum Theory and Thermodynamics, and so further developments on this subject could highlight aspects of "quantum gravity".

VIII. ACKNOWLEDGEMENTS

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